Sequenced Teaching of Problem Solving

HOME LEARNING MATERIAL

Year 3

Introduction

The modern maths curriculum in schools places a great focus on children's ability to solve problems and reason mathematically. When learning maths, children must be able to apply the core skills they have learned to a variety of problems and challenges.

At STOPS, we have devised 8 key problem-solving strategies that will help children approach problems with confidence. For each strategy, we have a range of problems that increase in difficulty so that children learn to tackle any tough maths problems with confidence.

Your child uses the STOPS problems in school and this book is designed to support and supplement the work that they are doing in school.

The problems and guidance in this book are designed to run in parallel to the work that your child is doing in school, enabling you to support them to become great mathematicians and successful problem-solvers - ensuring them success all the way to the end of primary school.

The STOPS Problem Solving Strategies:

A. Act it out / make a model

A great way to start solving problems. Act out, make or draw what the problem shows and you will be well on the way to solving it.

B. Trial and Error

A strategy every child must have - simply make some guesses and see how they go. Much better than not knowing how to start.

C. Trial By Improvement

The next step. Make an estimate, get a solution. Is it correct? Why not? How can we change our estimate to improve it? Children become more systematic.

D. Make a list or table

Many problems can be tackled by making a list of potential solutions. This can go hand-in-hand with strategies B and C to give children serious mental tools with which to solve tricky problems. Later, turn your list into organised tables and you can solve anything.

E. Find the pattern

Many problems can be solved by identifying a repeating pattern in shapes or numbers and using it to predict what may happen in other situations.

F. Simplify the problem

Some problems can be intimidating for children, but by making it more simple, it becomes more accessible.

G. Work Backwards

Start at the end and work back. Children will refine their skills of reasoning and 'inverse operations' to work their way through maths problems with ease.

H. Solve algebraically

It sounds more difficult than is, especially to children. When broken down into manageable steps of learning using shapes, symbols and eventually letters, children will become confident and experts in using algebra to solve problems

Using this booklet and how to best help your child

Each strategy has a one main problem to work through with your child and one other supporting problem. There are different steps within each strategy that make sure the problems are age-appropriate for your child. Remember that problem solving skills are very different to maths skills and children can develop at very different rates.

Each problem has notes afterwards that will give you guidance and examples of questions or modifications to support them if they are not sure or questions that could extend them if they are finding it easy.

Each problem is based on the STOPS problem-solving grid, where each strategy has 7 steps of difficulty for each of the 8 strategies. At the top of each problem in this book is the 'step' that the problem comes from, so that you can pick up on the next step of the strategy in every school year.

Some general tips:

- Encourage your child to make mistakes and feel positive about them, this is the only way to learn.

- Encourage children to record their thoughts in writing, on paper or in a special 'problem solving' notebook.

- Allow children time to think through for themselves, do not be tempted to do too much for them.

How do ‘steps’ and year groups work?

The STOPS problem solving skills are based on our famous grid, where each of the strategies has 7 steps within it that must be completed to be an expert problem-solver.

Each strategy is different, so ‘step 1’ does not always mean ‘year 1’.

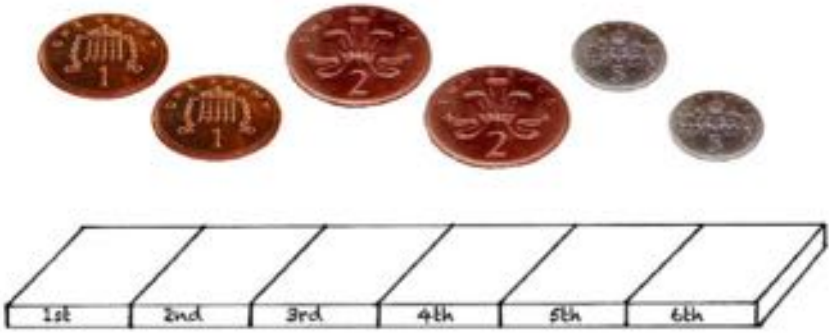
Below is our grid, with the recommended year 3 problems highlighted. This book will provide support and companion problems to the year 3 set of problems that your child is studying at school.

STRATEGY A - ACT IT OUT/MAKE A MODEL

STEP 6 - Count The Coins

Put the coins in the right order using the clues.

The first coin is double the value of the last coin.
The total of the first three coins is 27p.
The total of the last three coins is 13p.



Between the two 5p coins there is 1 coin.
Between the two 1p coins there are 2 coins.
Between the two 2p coins there are 3 coins.

How to help

- *Even at year 3, children will enjoy setting this activity up with coins, either real or toy versions.*
- *Encourage children to draw on their knowledge of Trial and Improvement strategies to find a few solutions*
- *For the addition in the first part, children could use either mental or written strategies.*
- *In part 1, use the first clue to establish that it must be 1p/2p, 5p/10p, or 10p/20p and fit the other coins in afterwards.*

Solutions:

- *For the first part, the solution is:*

20p, 5p, 2p, 1p, 10p

For the second part, one solution is:

1p, 2p, 5p, 1p, 5p, 2p

STEP 4 - Button Numbers

You have 21 buttons.
You have to make a 3 digit number using the buttons.
You must use all 21 buttons for each number.

HUNDREDS	TENS	ONES
		

Can you make 5 different numbers using the 21 buttons?

What is the largest number you can make?

What is the smallest number you can make?

How to help:

- *Set up a 3-column place value grid with the headings: hundreds, tens, ones/units.*
- *Find 21 buttons or counters.*
- *For the first part, allow children plenty of time to explore. 3 digit numbers are a fairly new concept at year 3 so allow children to make some numbers and say them.*
- *If you have a whiteboard or workbook, encourage children to write numbers in words, helping them to say and spell them correctly.*
- *Begin to compare numbers by saying which is bigger and smaller.*
- *For the second part, children may use reasoning or trial and error strategies. They must put the largest number in the hundreds column, so this would be a 9. They will have 12 buttons left so should put 9 in the tens column, leaving 3 in the ones/units.*
- *For the final part, children will want to put the smallest number in the tens. However, if you put 1 button in the hundreds column, you are left with 20 buttons. You cannot put 10 in a column, so 1 cannot be the number in the hundreds column. It will be a useful teaching point if children try to put 10 buttons in one column, explain that we 're-group' these 10 into 1 in the next column.*
- *To make it harder, use fewer buttons. Ask children to make 5 different numbers and write them in order.*
- *To make it easier, use more buttons up to 27 or reduce to 2 digit numbers.*

Solutions:

- *Part 1 has a large number of solutions. Check that all 21 have been used and that no columns have more than 10 buttons.*
- *Part 2: 993*
- *Part 3: 399*

STRATEGY B - TRIAL AND ERROR

Step 4 - Piles of gold coins!

Dalton has 25 gold coins.

He puts them in four piles.

The first pile has two more coins than the second pile.

The second pile has two more coins than the third.

The fourth pile has double the amount of coins as the second pile.

How many coins did Dalton put in each pile?

How to help:

- *This strategy focuses on 'Trial and Error, so allowing children to make mistakes is a key element of the strategy. Explain this to children and encourage them not to give up.*
- *Use counters or buttons to help model the problem if needed - and for fun! Children can just use pencil and paper to model the problem if they are confident.*
- *Children can pick a starting number and work through the clues, e.g. pick 10 as the start, therefore the second pile must be 12 and so on.*
- *Encourage children to check all the clues when they think they have found a solution.*
- *If children find it hard, give them the number for the first pile or use fewer coins in 3 piles, e.g. 10 coins in three piles.*
- *If children find it easy, increase the number of coins and piles, e.g. 32 coins in 6 piles.*

Solutions:

There are 7 coins in the first pile.
There are 5 coins in the second pile.
There are 3 coins in the third pile.
There are 10 coins in the fourth pile.

Step 4 - Baseball

Josh, Roman, Luca and Lisa play for a baseball team.

In one game, Josh scored 40 points, which was as many as Roman, Luca and Lisa together.

Roman scored 11 more than Lisa and 15 more than Luca.

What were Roman's, Luca's and Lisa's scores?

Challenge:

At the end of the game, the team had scored 2 more than last week's score and 5 more than the previous week. The total of the 3 weeks scores was 35. What were the scores each week?

How to help:

- *Once again, children should approach this with a 'Trial and Error' strategy. They should be used to using this now and remind them of times when they have used it before.*
- *Setting out information in a table is very useful if children are confident enough, e.g. below. If not, use jottings to keep track of unsuccessful trials.*

Josh	*Roman*	*Luca*	*Lisa*

- *Pick a number that could be a trial for Lisa, e.g. 5. Josh has scored 11 more than her, so he must have scored 16. This means that he has scored 15 more than Luca, so Luca must have 1. The total of these is 22, which is not enough.*
- *Ask children how to change their trial if they got an answer that was too big or too small.*
- *For the challenge, help children to see that three numbers must add to make 35, for example:*

___ + ____ + ____ = 35

- *Pick a starting number for this week's score. Last week's score was 2 below this and the previous week was below. An example trial using 10 as this week;'s score would be:*

5 + 8 + 10 = 23

23 is too low so children can try again with a higher first trial.

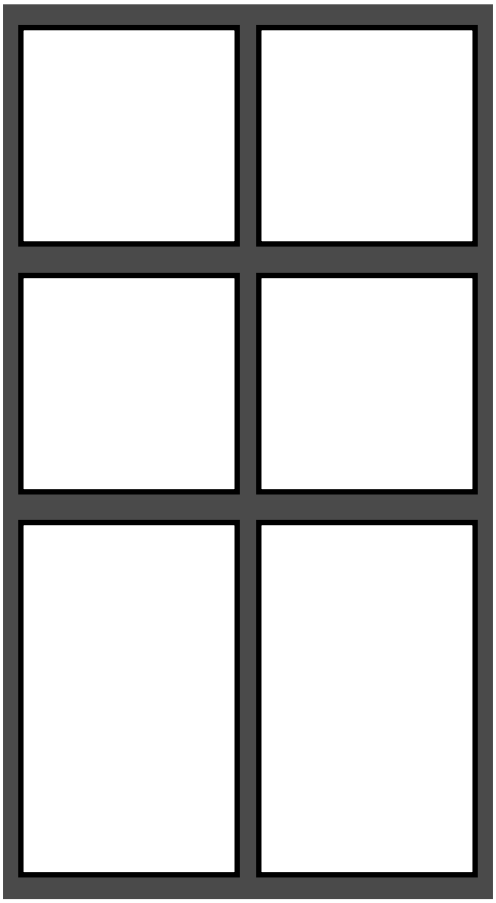
Solutions

Roman scored 22. Luca scored 7. Lisa scored 11.

Challenge: 14, 12 and 9 points. 9 + 12 + 14 = 35

Step 5 - Glass Window

Colour the shapes in this glass window by following the clues.

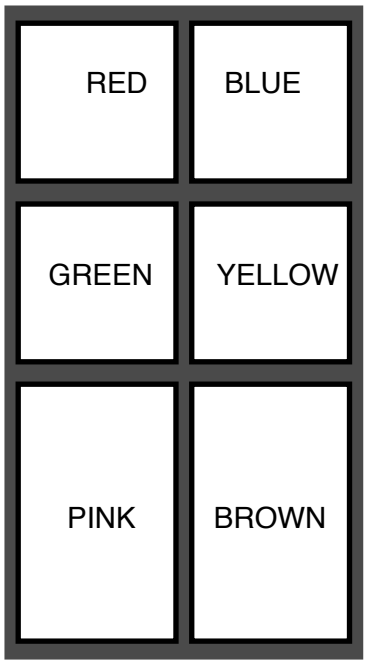


Red is not by the side of yellow but is by the side of blue
Green is between red and pink
Pink is not a square
Brown is underneath yellow
Red is on the left side

How to help:

- *This strategy focuses on a Trial-and-error strategy, so children can be encouraged just to make a trial,eg guess where red could be, then testing other colour locations against the clues.*
- *The problem can be solved by 'working backwards' to solve the clues in a systematic way.*
- *If children find it hard, give them a starting colour position, e.g. red.*
- *If children find it easy, ask them to design their own colour pattern and set clues for you to solve it.*

Solution:



Step 5 - Pick some penguins!

Penguin A + B together weigh 13kg.

Penguin A + C together weigh 14kg.

Penguin B + C together weigh 17kg.

What is the weight of each penguin?

How to help:

- *Encourage children to guess at some values first, in line with the trial-and-error strategy.*
- *Children should spot that the trials for penguins A and C should total 14kg.*
- *Children should use jottings to record trials, or even a systematic table if they are confident.*
- *Children could pick an initial trial, e.g. A = 10 and B = 3, they can then use the other clues to derive C. In this example, if A = 10, C must be equal to 4 if A + C = 14. In this case, B + C must be 3 + 4 = 7. This cannot be correct because the clue tells us that B + C = 14, so we must start with another trial.*
- *If children find it hard, give them a number range for A, e.g. it is between 4 and 7, or give them the value of A.*
- *If children find it easy, substitute higher number values for A, B and C and adjust the clues accordingly.*

Solution:

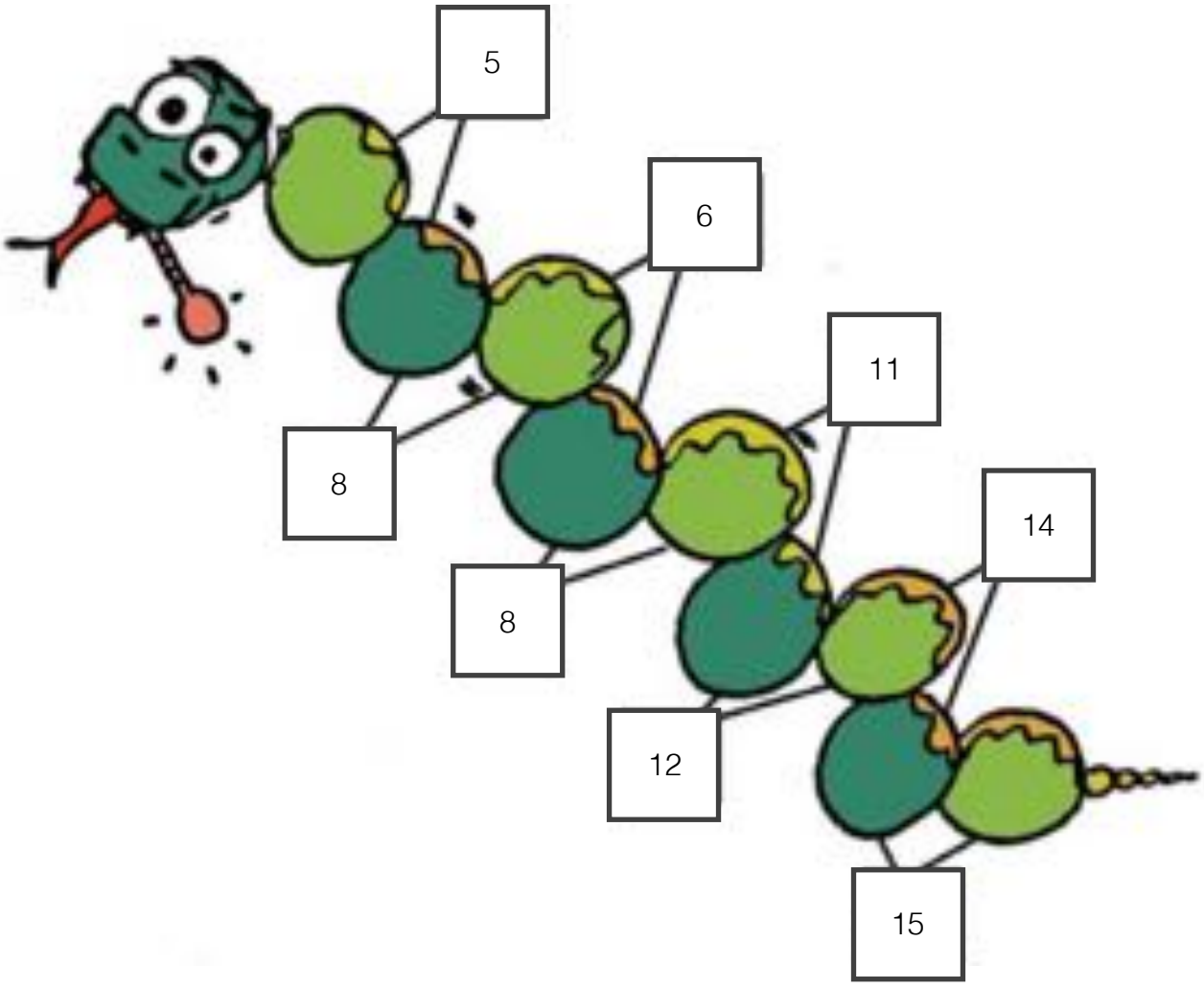
A = 5
B = 8
C = 9

STRATEGY C - TRIAL BY IMPROVEMENT

Step 4 - Number snake

Place the digits 1 - 9 in the circles once each.

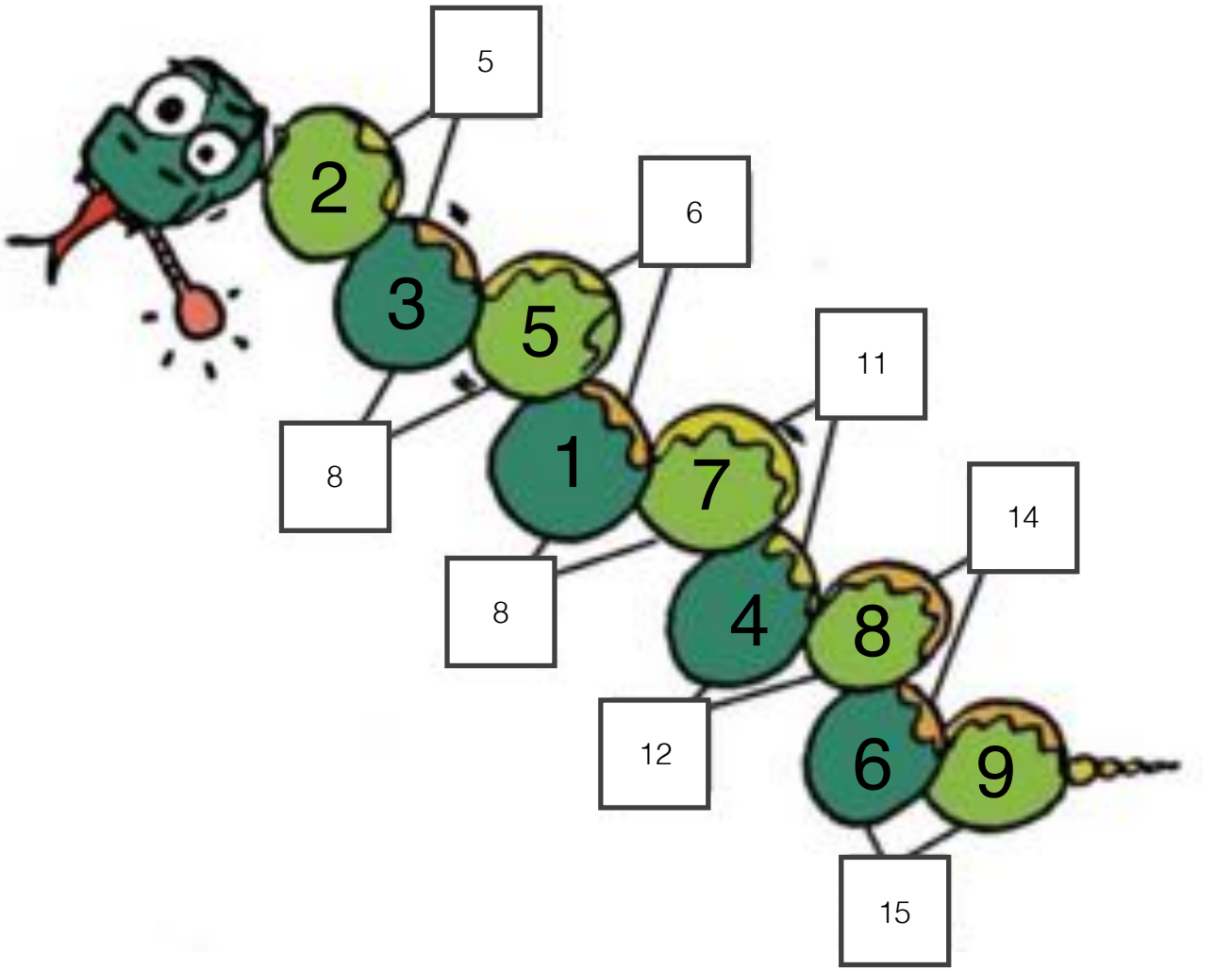
The numbers in the two circles must add up to the number in the square that links the two circles.



How to help:

- *This strategy is focus on trial-by-improvement, so children can make trials and then adjust them as they go based on estimation and reasoning, rather than just 'guessing.'*
- *Identify where the totals in the squares are larger and ask children what that might tell them about where to put the digits 1-9.*
- *Children will use inverse operations to establish what is needed to go next to a number in a circle to give the correct difference.*
- *At some point, the trial will not work and children will have to adjust by swapping numbers, or even starting again*
- *If children find it difficult, give them one or two starting numbers*
- *If children find it easy, ask them to create one for you to solve.*

Solutions:



Step 4 - Number detective

This is Detective Oswald Mallory.

He has found a two-digit number less than 50. The sum of its digits is 8. The difference between the digits is 2.

What number did he find?

Next, he found a two-digit even number.
One of its digits was double the other.
The number was greater than 50 but less than 100.

What number did he find?

How to help:

- *Ensure that children know about odd and even numbers. Ask them to explain the difference and give examples.*
- *Children will also need to know about sum and difference, as well as doubles.*
- *Children can begin by guessing numbers and testing against the clues.*
- *To work more systematically, ask children to make a list of all the numbers whose digits could total 8. You can then use this list to test each number against the clues.*
- *The same would work in part 2, make list of possible numbers where one digit is double the other.*
- *If children find it hard, create the list of possible numbers for them, explaining how you are doing so and how you know.*
- *If children find it easy, ask them to make clues for a number that you have to find. They have to make sure that the clues only describe one number and not several.*

Solutions:

Part 1 - 35
Part 2 - 84

STRATEGY D - MAKE A LIST OR TABLE

Step 3 – Apples on the Tree

An apple tree has more than 30 and less than 50 apples.

Aisha counts them in threes and has 1 left over.

She counts them in fives and has 2 left over.

How many apples were on the tree?

How to help:

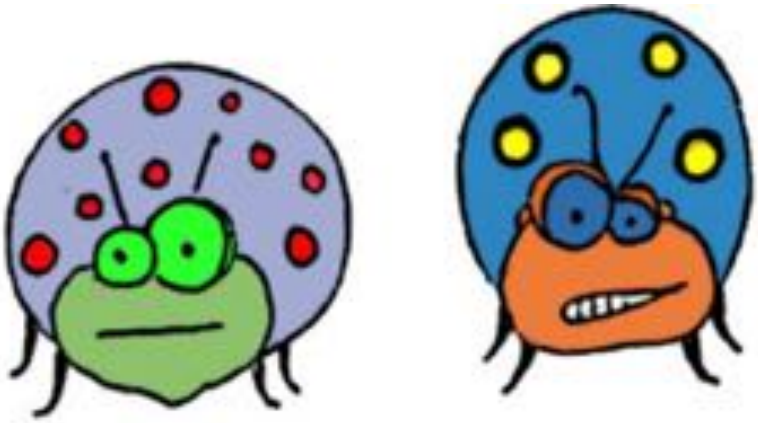
- *This strategy requires children to find a systematic way of recording their work in a list or table.*
- *Ask children what we can make a list of that might help us with this.*
- *Children may suggest that we could use trial and error to guess at numbers and divide by 3 and 5. This is a valid strategy - praise them - but we are going to try to use lists and tables to help us with this problem.*
- *Write out the multiples of 3 together. Ask children what numbers would have 1 left over when divided by 3. An example would be 7 - you could count 2 groups of 3 and have1 left over - show this with counters if needed.*
- *Write together the multiples of 5 and ask what number would have 2 left over if we divided by 5. An example would be 17.*
- *Ask children to look at the two lists and find a number that is 1 more than a multiple of 3 and 2 more than a multiple of 5. Children should work through the list systematically until they can find this number.*
- *It may be useful to also record two more lists of numbers that are 1 more than multiples of 3 and 2 more than multiples of 5, if children are confident enough.*
- *If children find it hard, model drawing the lists for them and simple examples until they are ready to move on.*
- *If children find it easy, ask for another number of apples greater than 50 that would also work*

Solutions:

There are 37 apples.

If there were more than 50 apples, 67 would also be a solution.

Step 3 – Spotty Aliens!



Purple aliens have 9 spots.

Blue aliens have 4 spots.

Altogether, a group of purple and blue aliens have 65 spots.

How many of each type could there be?
Is there more than one solution?

Challenge:

How many of each type of alien if there are 90 spots?

How to help:

- *Ask children if they have used lists or tables before and if we could make lists to help us solve this problem.*
- *Children should make lists of multiples of 9 and multiples of 4. The solution must be from these lists. Note that the 9 times table may be tricky so support them to list the multiples of 9.*
- *Then, they can try to randomly pick pairs of numbers from each list to see if they can make a total of 65. Ask children how they will record their trials so that they don't do a repeat.*
- *At year 3, children can use mental or written methods for addition.*
- *If children find it easy, encourage them to find more than one solution or move on to the challenge.*
- *If children find it hard, tell them how many purple aliens there are in the solution as a start.*

Solutions:

There would be 5 purple aliens (45 spots) and 5 blue aliens (20 spots).
A second solution would be 1 purple alien (9 spots) and 14 blue aliens (56 spots)

Challenge:
Several solutions are possible:
10 purple aliens (90 spots) and no blue aliens.
6 purple aliens (54 spots) and 9 blue aliens (36 spots)
2 purple aliens (18 spots) and 18 blue aliens (72 spots)

STRATEGY E - FIND THE PATTERN

Step 3 – Basketball tournament

In a basketball tournament, there are three teams. Each team must play all the other teams once only?

How many matches will be played?

How many matches will be played if there are 4 teams?

How many matches will be played if there are 5 teams?

How to help:

- *Ask children how making a list or modelling the problem might help with this strategy.*
- *Explain that we might be able to find a pattern that will help us solve the problem.*
- *Encourage children to make up three teams - A, B and C (or whatever team names they want!)*
- *Write out that:*
  A must play B
  A must play C
  B must play C
- *Check that there are no more teams to play each other and decide that there are 3 matches.*
- *Repeat for the examples of 4 and 5 teams.*
- *Show children how to be systematic, i.e. write all of A's matches, then B etc.*
- *If children find it easy, extend to 6 or 7 teams.*

Solutions:

For 3 teams, there are 3 matches.

For 4 teams, there are 6 matches.

For 5 teams, there are 10 matches.

If you extend to 6 teams, there would be 15 matches.

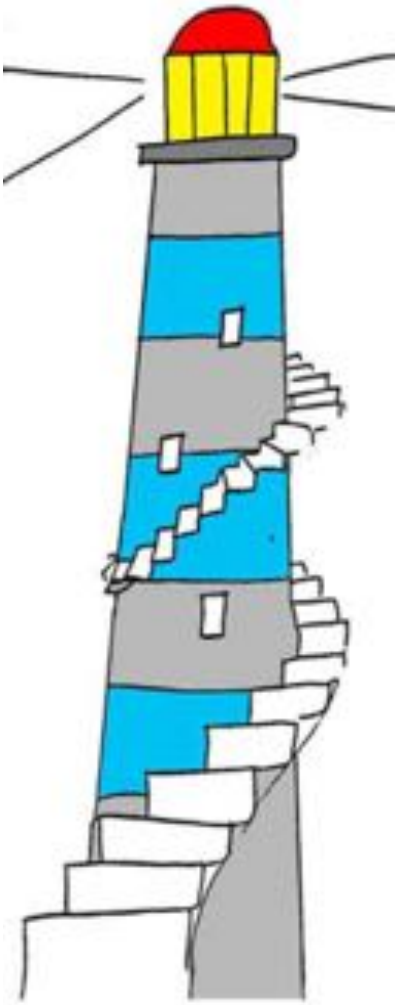
Step 3 – To the lighthouse!

There are 99 steps that lead to the top of the lighthouse.

Roman starts at the ground and jumps 5 steps at a time.

Aisha starts on the 21st step and jumps 3 steps at a time.

Josh starts on the 64th step and jumps 2 steps at a time.



Who reaches the top of the lighthouse first?

How to help:

- *Discuss how to solve the problem with the child, what could their strategy be?*
- *Explain that each child jumps at the same time.*
- *Writing out the multiples of 5 from 0 is a good starting point, it shows us that Roman reaches the top after 20 jumps.*
- *Next to this, count up in 3's from 21. We then see that Aisha has reached the top after 27 jumps.*
- *Finally, count up in 2's from 64 and we can see that Josh reaches the top in 18 jumps.*
- *If children find it hard, reduce the steps to 30 and adjust the clues accordingly, e.g. Aisha starts at 6 and Josh starts at 10.*
- *If children find it easy, repeat with different multiples, e.g. of 6 and 7.*

Solutions:

Josh wins, he gets to the top in 18 jumps, just beating Roman who gets there in 20 jumps.

STRATEGY F - SIMPLIFY THE PROBLEM

Step 3 – What's for lunch?

John, Lisa and Luca have all brought a different lunch.

Sandwiches:
They have ham, cheese or chicken sandwiches.
John does not like chicken. Luca always brings ham.

Drinks:
They have orange juice, milk or water.
Lisa cannot drink milk or juice.
Luca does not have juice.

Fruit:
They have a banana, an apple and a pear.
John brings an apple every day.
Lisa never eats bananas.

What does each person bring for lunch?

How to help:

- *The point of this strategy is to take a problem with lots of parts and simplify. Ask children how we can break this problem down into smaller parts.*
- *First, look at the sandwiches. Children should make jottings to record, for example making a heading of 'sandwiches' and writing the three names.*
- *We can see that John does not like chicken - ask children what this tells us? It means that he must have ham or cheese.*
- *We can also see that Luca always brings ham, so we can write ham next to his name.*
- *If John does not like chicken, he must have the cheese, so we can write cheese next to his name and ham next to Luca.*
- *Repeat for the rest of the parts of the lunch.*
- *If children find it easy, allow them to complete independently.*
- *If it is too difficult, use pictures or actual food and drink to model the problem in real life. You could use a trial and error strategy to guess at foods and see if they satisfy the clues.*

Solutions:

John has a cheese sandwich, orange juice and an apple.
Lisa has a chicken sandwich, water and a pear.
Luca has a ham sandwich, milk and a banana.

Step 3 – Our favourite toys

Josh, Aisha and Luca are friends.

They each have favourite toys: a teddy, a car and a boat.

Josh said, "Each of us likes only two of the toys."

Aisha said, "Each of the toys is liked by only two of us."

Luca said, "Josh likes the car and I don't like the teddy.

Which two toys do each of the children like?

How to help:

- *This is a really interesting logic problem and children will most likely need a table to record the solutions.*
- *Check children understand the clues: each toy has two children that likes it and each chid likes two toys.*
- *Re-read the clues. If Luca does not like the teddy, which two toys does he like? He must like the car and the boat.*
- *If Josh likes the car, and Luca likes the car, Aisha must be the one that does not like the car.*
- *Show children how to use a simple logic table, if they are confident:*

	Teddy	**Car**	**Boat**
Josh		✓	
Aisha			
Luca	x	✓	✓

- *The table can be used to record that Josh likes the car, Luca does not like the teddy, therefore likes the car and boat. Now we can see that Aisha does not like the car, so she must like the teddy and boat.*
- *If children find it too difficult, use a trial and error strategy to find solutions and test against the clues.*

Solutions:

Aisha likes the teddy and the boat.

Luca likes the car and the boat.

Josh likes the teddy and the car.

STRATEGY G - WORK BACKWARDS

Step 4 - Don't be late for school!

Josh's sleepy cousin was trying to decide when to get out of bed.

- He needs 35 minutes to wash and shower.
- It will take him 15 minutes to eat his breakfast.
- He needs 20 minutes to walk to school.
- He needs to be at school by 8:00am.

What time should he get out of bed?

Challenge: if it takes him 25 minutes to walk home from school and he gets home at 4:10, what time did school finish?

How to help:

- *As adults, we might add up all of the smaller time intervals and count that back from 8:00am. If children are confident enough to see this strategy, by all means let them continue.*
- *There is a lot of value to be gained by working backwards from each clue, both in terms of problem solving skills and skills with calculating time.*
- *This is a good opportunity to check on children's understanding of telling the time and calculating intervals of time, e.g. how long is it between 7:50 and 8:05am?*
- *Start with 8:00am and count back 20 minutes. Children will need to know that we start counting at 60 because there are 60 minutes in the hour.*
- *Continue working backwards with the clues until you have the solution.*
- *If children find it difficult, replace the times with 15 and 30 minute intervals.*
- *If children are confident, change times to multiples of 1 minute, e.g. it takes him 17 minutes to walk to school.*

Solutions:

He should wake up at 6:50am.

Challenge - he left school at 3:45pm.

Step 3 – Missing numbers...

Josh is doing some calculations but some numbers are missing! Can you find the missing digits to make these calculations correct?

	4	2	5
+		3	
	5		7

	2		3
+	5	2	7
		9	

	4	3	
-		3	4
	3		1

	8	5	0
-		3	
	3		9

How to help:

- *Ensure that children are confident with basic column addition and subtraction by practising a few examples.*
- *Identify that the top two are are addition, the bottom two are subtraction.*
- *Ask children to explain their thoughts and reasoning as they work through the problems.*
- *If children are not sure, model one yourself, explaining what you are doing as you go.*
- *If children are finding it easy, create some examples with greater numbers or with column multiplication.*

Solutions:

	4	2	5
+	1	3	2
	5	5	7

	2	6	3
+	5	2	7
	7	9	0

	4	3	5
-	1	3	4
	3	0	1

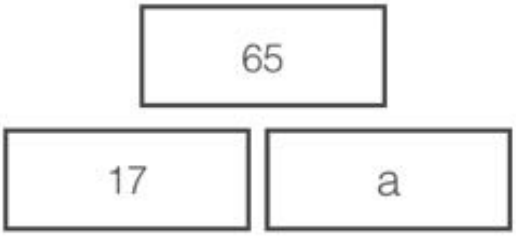
	8	5	0
-	5	3	1
	3	1	9

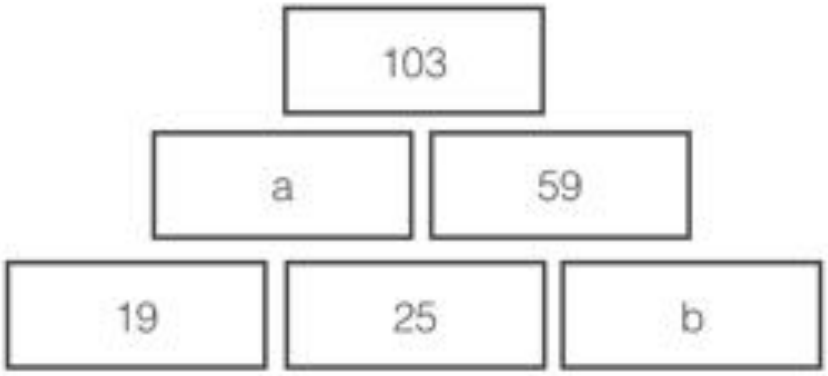
STRATEGY H - SOLVE WITH ALGEBRA

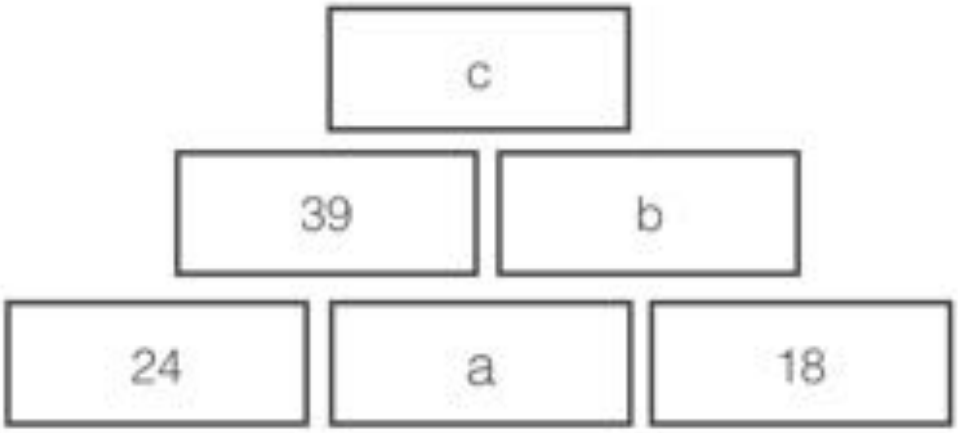
Step 3 – Pyramids

In these pyramids, each number is equal to the sum of the two numbers below it.

Find the missing numbers in each pyramid.



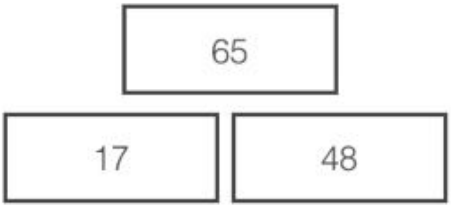


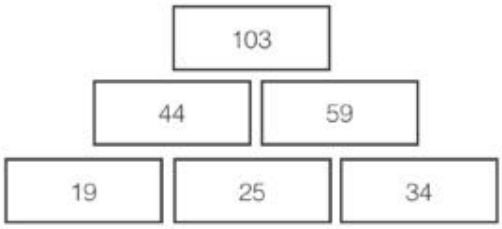


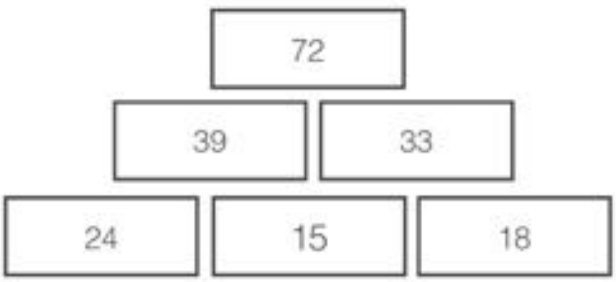
How to help:

- *Children will make the jump here from using pictures or shapes to represent missing numbers to using letters, so take time to explain this.*
- *Children will also need to be confident at knowing about the inverse relationship between addition and subtraction. Test theme with examples such as 25 + ? = 42.*
- *Ask children where the missing numbers are and how to begin the problem.*
- *If children are not sure, write the expressions away from the pyramid, using a blank space or question mark for the missing number. For example: 17 + ___ = 65. Show how we can re-write this as a subtraction to find the solution, e.g. 65 - 17 = ____.*

Solutions:







Step 3 - Burger and Chips

A burger and chips costs £1.60

A burger and two boxes of chips costs £2.10.

What does a box of chips cost?

What does a burger cost?

How to help:

- *It is certainly possible to express this problem as two equations, for example:*

  b + c = £1.60

  b + 2c = £2.10 (or b + c + c = £2.10)

This is a useful exercise for children to model real-life situations using algebra. However, rearranging these equations to find the solution is way beyond the year 3 curriculum.

- *When these formula are established, you can easily use trial and error, along with recording in a table to establish what the correct values may be.*
- *Ask children what the difference is in the equations - an extra box of chips. Establish that this has cost 50p in the total, so the chips must cost 50p.*
- *If the chips cost 50p, subtracting this from £1.60 shows that the burger must cost £1.10.*
- *If children find it hard, stick with a trial and error strategy and use pictures to represent the burger and chips in your recording table.*
- *If children are confident, give some different expressions with the same values for b and c. For example, 2b + 2c = ? or 10c + 2b = ?. Ask children to write in words what each expression is showing, for example 2b + 2c is the cost of two burgers and two portions of chips.*

Solutions:

The chips cost 50p.
The burger costs £1.10.

Well done - you have solved all of the year 3 logic and reasoning problems in this booklet and you are an expert problem-solver!

Head over to www.stopsproblemsolving.co.uk and check out the COVID19 zone for more games and problems.