Roomy boxes

Children cut squares from a square piece of paper, fold up the sides to form an open cuboid and find out which size will hold the most cm³ cubes.

Skills practised:

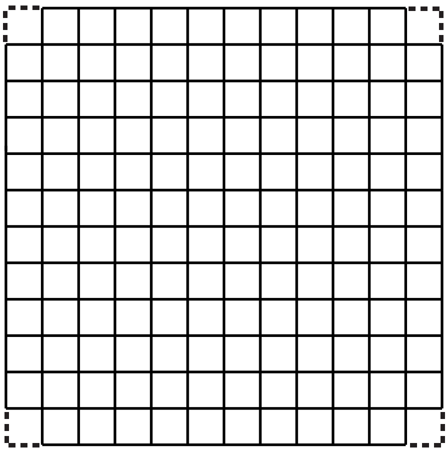
- Finding volumes of cuboids
- Multiplying three numbers together
- Recording results in a table

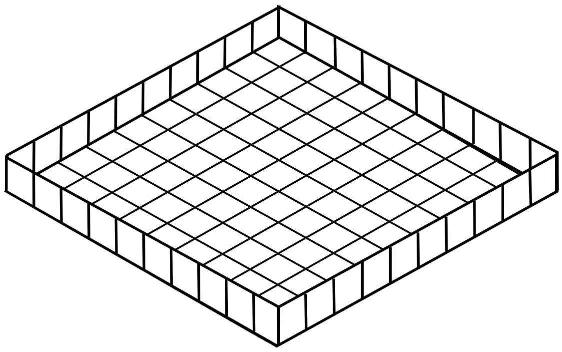
Conjecture: *The cuboid which will hold the greatest volume by taking squares out of the corner of a square piece of paper and folding the resulting net, will be an open cube.*
(Note to teachers: This is actually false! Your children might like to prove it to be wrong!)

What to do:

Children work individually or in pairs.

1. Cut out a 12cm by 12cm square from a sheet of cm^2 paper.
2. Cut a square centimetre from each corner.





3. Now fold it to form an open cuboid.
4. Work out how many $1cm^3$ cubes this box could hold.
5. Now cut a larger square from each corner so that the missing piece is a 2cm by 2cm square. Fold the sides up again to form an open cuboid. Work out how many $1cm^3$ cubes this box could hold.
6. Repeat, so that this time the missing piece from each corner is a 3cm by 3cm square.
7. Keep on going. Record your results in a table.
8. Which box could hold the greatest number of $1cm^3$ cubes?

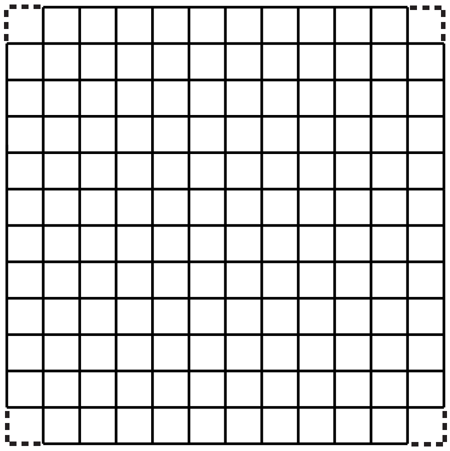
Try starting with other size squares, e.g. 15cm by 15cm and then 20cm by 20cm. Can you predict which cuboid will hold the greatest volume of $1cm^3$ cubes? Instead of cutting squares out with whole number of cm sides, you could try cutting out squares with lengths, ½cm, 1cm, 1½cm, 2cm, 2½ cm... You might like to draw line graphs to show your results, with the height of the cuboid on the x-axis and the column on the y-axis. Before you do, what shape you think the line graph will be?

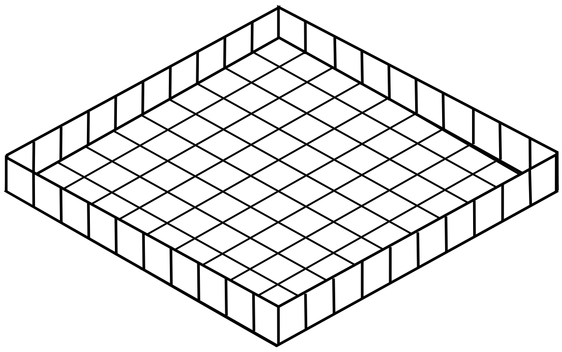
Aim:
- To make and test predictions
- To decide how best to records results

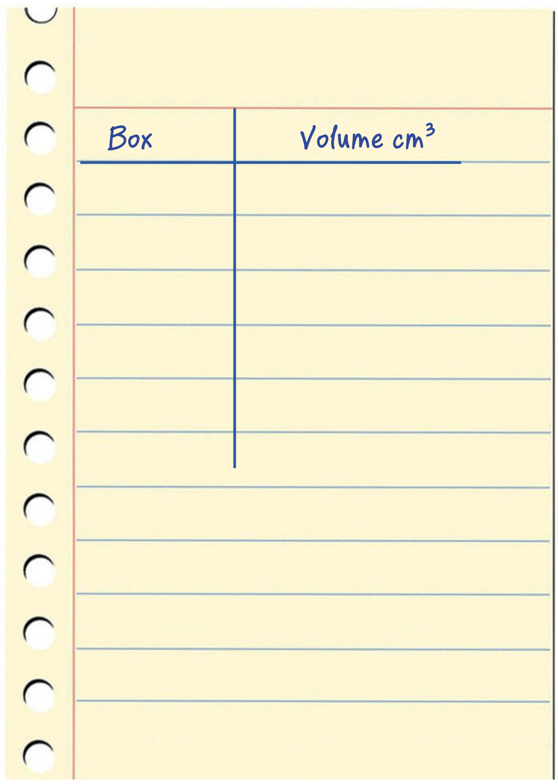
Minimum number of calculations expected
12

Roomy boxes

1. Cut out a 12cm by 12cm square from a sheet of cm^2 paper.
2. Cut a square centimetre from each corner. Fold it to form an open cuboid.







3. Work out how many $1cm^3$ cubes this box could hold.
4. Now cut a larger square from each corner so that the missing pieces are 2cm by 2cm square. Fold the sides up again to form an open cuboid. Work out how many $1cm^3$ cubes this box could hold.
5. Repeat, so that this time the missing piece from each corner is a 3cm by 3cm square.
6. Keep on going. Record your results in a table.
7. Which box holds the greatest number of $1cm^3$ cubes?

Try starting with different size squares, e.g. 15cm by 15cm and then 20cm by 20cm. Can you predict which cuboid will hold the greatest volume of $1cm^3$ cubes? Instead of cutting squares out with whole number of cm sides, you could try cutting out squares with lengths, $\frac{1}{2}$cm, 1cm, $1\frac{1}{2}$cm, 2cm, $2\frac{1}{2}$cm and so on. Draw line graphs to show your results, with the height of the cuboid on the x-axis and the volume on the y-axis. Before you do, what shape do you think line graph will be?

Queued cubes

Children apply a combination of knowledge of 3D shape, area and volume to solve a problem that introduces surface area.

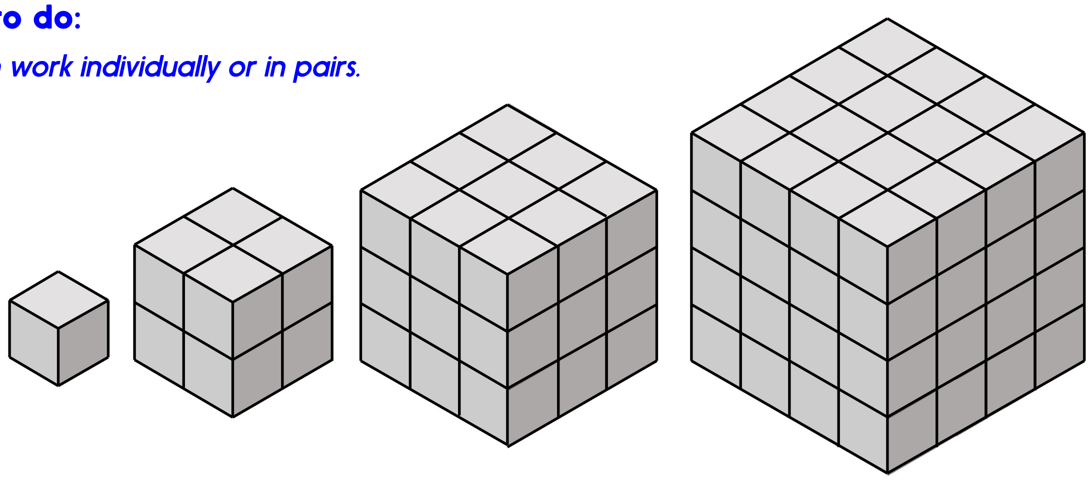
Skills practised:

- Applying knowledge of 3D shape: nets of cubes
- Calculating area of rectilinear shapes and volume of cuboids
- Generalising relationships between numbers

Conjecture: *Doubling the length of the sides of a cube increases the surface area by a factor of 4 and the volume by a factor of 8.*

What to do:

Children work individually or in pairs.



1. Imagine covering a 1x1x1cm cube in wrapping paper (with no tabs or overlaps). Now visualise peeling off the paper to leave the *net* of this shape.
   a. What is the area of this net? This is the **surface area** of the cube. We'll call it 'area **1**'.
   b. What is the volume of this shape? We'll call it 'volume **1**'.
2. Now imagine a 2x2x2cm cube.
   a. What would be the surface area of this shape? Let's call this 'area **2**'.
   b. What is the volume of this shape? We'll call it 'volume **2**'.
   c. What fraction of area **2** is area **1**?
   d. What fraction of volume **2** is volume **1**?
3. Go through the same process with a 3x3x3cm cube.
   a. Can you predict the surface area of this shape: 'area **3**'? Now calculate it to find out if you were right.
   b. Can you predict the volume of this shape? Calculate it to find out if you were right.
   c. What fraction of area **3** is area **1**? What fraction of volume **3** is volume **1**?
4. Repeat this for a 4x4 cube. What fraction of area **4** is area **1**? What fraction of volume **4** is volume **1**?
5. You'll be spotting some patterns and relationships between the numbers by now. Can you write about any patterns you've noticed?
6. If you were given a cube with 10cm sides, would you be able to quickly calculate its surface area? What about a cube with sides of any length: ***n*** cm ?

HINT: Organising your results in some way will be really helpful. Think about what you do to specific numbers when beginning to make *generalisations* for any numbers in a sequence.

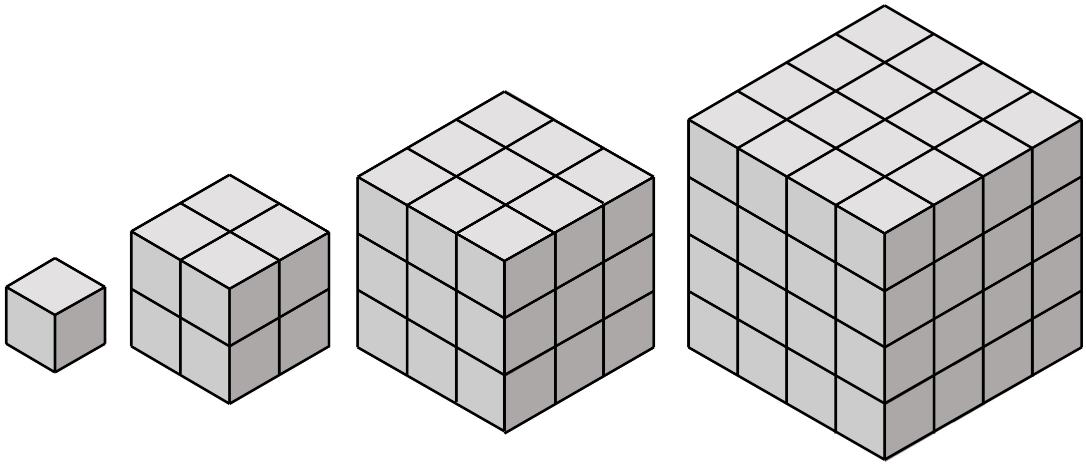
Aims:

- To apply knowledge of area and volume
- To begin to generalise a term in a sequence using ***n*** to stand for the number of the term in a sequence

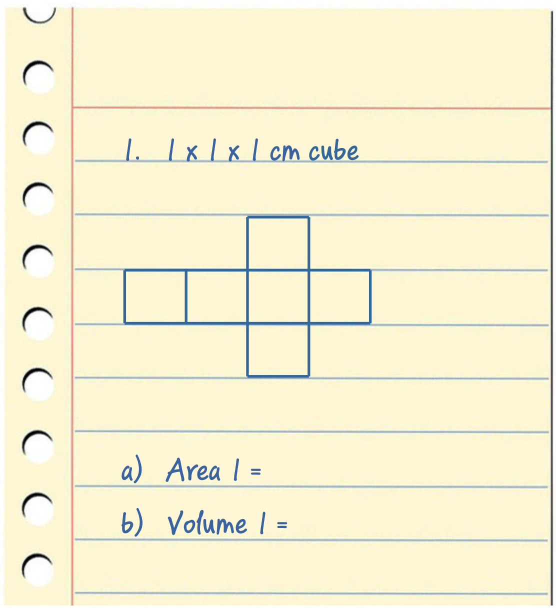
Minimum number of calculations expected

15

Queued cubes



1. Imagine covering a 1x1x1cm cube in wrapping paper (with no tabs or overlaps). Now visualise peeling off the paper to leave the *net* of this shape.
   a. What is the area of this net? This is the **surface area** of the cube. Let's call it 'area **1**'.
   b. What is the volume of this shape? Let's call it 'volume **1**'.



2. Now imagine a 2x2x2cm cube.
   a. What would be the surface area of this shape? Let's call this 'area **2**'.
   b. What is the volume of this shape? Let's call it 'volume **2**'.
   c. What fraction of area **2** is area **1**?
   d. What fraction of volume **2** is volume **1**?
3. Go through the same process with a 3x3x3cm cube.
   a. Can you predict the surface area of this shape: 'area **3**'? Now calculate it to find out if you were right.
   b. Can you predict the volume of this shape? Calculate it to find out if you were right.
   c. What fraction of area **3** is area **1**? What fraction of volume **3** is volume **1**?
4. Repeat for a 4x4 cube. What fraction of area **4** is area **1**? What fraction of volume **4** is volume **1**?
5. Can you spot some patterns and relationships between the numbers?Write about any patterns you've noticed.
6. If you were given a cube with 10cm sides, would you be able to quickly calculate its surface area? What about a cube with sides of any length: ***n*** cm ?